

B.Sc. Semester - 1 (NEP-2020) Examination
OCT/NOV-2025 (As Per Syllabus Year June-2023)
MATH: .Matrices and Coordinate Geometry (Major-1)

Time: 2:00 Hours**Marks:50****Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one (05)

- (1) Define: Diagonal Matrix, Triangular Matrix, Symmetric Matrix, Orthogonal Matrix, Echelon form of Matrix.
- (2) A^T and B^T be the transpose of A and B respectively then prove that $(AB)^T = B^T A^T$, where A and B being conformable to multiplication.

Que.1(B) Answer any one (05)

- (1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew symmetric matrix.
- (2) If A and B are given matrices such that $AB = BA$ then prove that $(AB)^n = A^n B^n$, where $n \in N$.

Que.2(A) Answer any one (05)

- (1) Define Rank of Matrix and show that rank of any Matrix $\geq$ rank of its every sub Matrix.
- (2) Prove that the rank of the product of two matrices cannot exceed the rank of either matrix.

Que.2(B) Answer any one (05)

- (1) Find inverse of Matrix $\begin{bmatrix} 5 & 8 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary row operations.

- (2) Find Rank of matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$

Que.3(A) Answer any one (05)

- (1) State and prove Cayley – Hamilton theorem.
- (2) Prove that there is Unique Eigen Value with respect to any Eigen Vector of given Matrix.

Que.3(B) Answer any one (05)

- (1) Define Augmented Matrix and prove that the equation $AX = B$ has a unique solution if A is non-singular.
- (2) Solve following linear equations if possible:

$$\begin{aligned}x + y + z &= -3 \\ 3x + y - 2z &= -2 \\ 2x + 4y + 7z &= 7\end{aligned}$$

Que.4(A) Answer any one (05)

(1) In usual notation derive the equation of a conic in polar co-ordinates

$$\frac{l}{r} = 1 + e \cos \theta.$$

(2) Find the equation of straight line passing through the points $\left(1, \frac{\pi}{2}\right)$ and $(2, \pi)$

Que.4(B) Answer any one (05)

(1) Derive area of the triangle in polar co-ordinates.

(2) Show that the sum of reciprocal of segments of any focal chord of conic $\frac{l}{r} = 1 + e \cos \theta$ is constant.

Que.5(A) Answer any one (05)

(1) Find Echelon form of Matrix $\begin{bmatrix} 1 & 0 & -4 \\ 0 & -1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$

(2) Obtain inverse of Matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ using Cayley – Hamilton theorem

Que.5(B) Answer any one (05)

(1) Find a and b such that the equations

$$x - 2y + 3z = 1$$

$$3x + y - z = 4$$

$$2x - 2y + az = b \quad \text{have (i) One solution} \quad \text{(ii) No solution.}$$

(2) Obtain the centre and radius of circle from the equation

$$r^2 - r(2\cos\theta + 3\sin\theta) = 0$$
